Online Appendix to

“The Aggregate Matching Function and Job Search from Employment and Out of the Labor Force”

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Not For Publication

1 The extended Kalman filter

This section describes the (first-order) extended Kalman filter (EKF). The EKF linearizes the non-linear observation or state equations and applies the regular Kalman filter recursions to this linearized system. Let the non-linear state space be described by the following measurement and transition equation:

$$y_t = h(\theta_t, x_t) + \epsilon_t, \tag{1}$$

$$\theta_t = f(\theta_{t-1}) + \eta_t, \tag{2}$$

where f and h are non-linear functions, y_t is a vector of observables and θ_t is a vector of unobserved states. Note that in the benchmark model, f is in fact linear. For the state-space system given in (1) and (2) the EKF recursions are the following:

$$\theta_{t|t-1} = f(\theta_{t-1|t-1}), \tag{3}$$

$$P_{t|t-1} = F_t P_{t-1|t-1} F_t' + Q, \tag{4}$$

$$Z_t = H_t P_{t|t-1} H_t' + R + H_t C + C' H_t', \tag{5}$$

$$\widetilde{y}_t = y_t - h(\theta_{t|t-1}, x_t), \tag{6}$$

$$K_t = (P_{t|t-1} H_t' + C) Z_t^{-1}, \tag{7}$$

$$\theta_{t|t} = \theta_{t|t-1} + K_t \widetilde{y}_t, \tag{8}$$

$$P_{t|t} = P_{t|t-1} - K_t (H_t P_{t|t-1} + C'), \tag{9}$$

where F_t and H_t are the Jacobian matrices of the transition and measurement equations, respectively:

$$F_t = \frac{\partial f}{\partial s}|_{\theta_{t-1|t-1}}, \tag{10}$$

$$H_t = \frac{\partial h}{\partial s}|_{\theta_{t|t-1}, x_t}. \quad (11)$$

2 Diagnostic tests

The Maximum Likelihood estimation procedure relies on the (normalized) forecast errors being identically and independently normally distributed. The forecast errors can be written as $\widetilde{y}_t = (B')^{-1}(y_t - h(\theta_{t|t-1}))$, where the notation follows that of the previous section and where $B'B = \mathbb{E}[\widetilde{y}_t \widetilde{y}_t']$.

The normalized forecast errors are tested for normality, homoscedasticity and serial correlation.[1] In all cases, the null hypothesis of normality, homoscedasticity and no serial correlation cannot be rejected at (at least) the 10 percent level of significance.

3 Robustness

This section of the Appendix reports several robustness exercises relating to the specification of the state-space system and the estimation procedure. Estimates of the matching elasticity and the estimated time-path of match efficiency under the different specifications are reported in Table 1 and Figure 1, respectively.

3.1 Different laws of motion for latent variables

The benchmark specification followed the literature on time-varying coefficients (see e.g. Cogley and Sargent, 2005; Primiceri, 2005) and assumed that the latent variables are random walks. This subsection shows that the results change very little when the unobserved components are assumed to follow AR(p) processes instead. In particular, I consider cases in which $p = 1, 3, 6$. Table 1 and Figure 1 show that the estimated matching elasticity and efficiency are very close to those recovered from the benchmark specification.

3.2 Covariance between observation and state innovations

To economize on estimated parameters, the benchmark model assumed that the correlation between innovations of the unobserved time-varying parameters and those in the observation equations was zero. This subsection relaxes the assumption. In particular, it is assumed that $\mathbb{E}[\epsilon_t \eta_t'] = C$, where $\epsilon_t = (\epsilon_t^T, \epsilon_t^U, \epsilon_t^I)'$ and $\eta_t = (\eta_t^U, \eta_t^E, \eta_t^I)'$. Table 1 and

[1] The respective tests are the Henze-Zirkler multivariate normality test (conducting the Jarque-Berra test for normality on each forecast error separately delivers similar results), an F-test of equal variances (based on the first and the last third of the sample) and the Ljung-Box test of serial correlation using up to six lags.

Table 1: Matching elasticity estimates under alternative specifications

	μ	
benchmark	0.765	(0.031)
AR(1)	0.729	(0.016)
AR(3)	0.799	(0.019)
AR(6)	0.789	(0.019)
$C \neq 0$ in $var([\epsilon_t, \eta_t]') = \begin{pmatrix} R & C' \\ C & Q \end{pmatrix}$	0.691	(0.027)
JOLTS total hires	0.588	(0.025)
4 equation model	0.719	(0.025)
2 equation model	0.789	(0.034)
non-CRS	0.746	(0.028)
trend	0.761	(0.020)

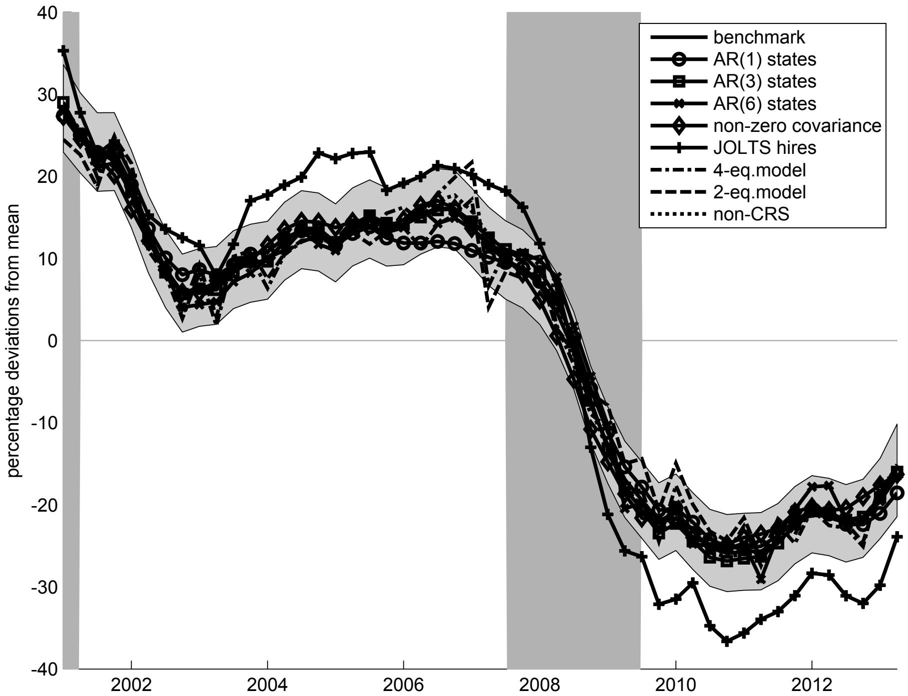
Notes: The table reports estimates of the matching elasticity under alternative specifications. "AR(1)", "AR(2)" and "AR(3)" refer to the case when the latent variables are assumed to follow AR processes with 1, 2, or 3 lags instead of being random walks as in the benchmark. "$C \neq 0$" refers to the case when the innovations to the unobserved states are allowed to covary with the measurement errors. "JOLTS total hires" replaces the CPS measure of hires with that from the JOLTS database. "4 equation model" and "2 equation model" model refer to the specifications in (3.4) and (3.5), respectively. "Non-CRS" refers to the case when the assumption of constant returns to scale is relaxed. "Trend" refers to the case when all latent variables are allowed to include a deterministic linear trend.

Figure 1 show that estimates of the matching elasticity and efficiency are very close to the benchmark values.

3.3 Alternative measure of total hires

The benchmark model uses the sum of all worker flows into employment from the CPS as a measure of hires. Alternatively, one could use the hires variable available in the JOLTS data. These two measures of total hires are not equal to each other. The CPS numbers are almost twice as large and almost half as volatile as the JOLTS hires data. Davis, Faberman, Haltiwanger, and Rucker (2008) find that a large part of this discrepancy can be attributed to a lack of establishment openings and very young establishments and to the treatment of non-respondents in the JOLTS data. Nevertheless, Figure 1 shows that the decline in match efficiency is even stronger during the Great Recession in this case. The matching elasticity is estimated to be somewhat lower at 0.59 (see Table 1).

Figure 1: Match efficiency estimates under alternative specifications



Notes: match efficiency estimates based on the 'benchmark" and alternative specifications: "AR(1)", "AR(3)" and "AR(6)" refer to the cases when the unobserved states are assumed to follow AR(1), AR(3) and AR(6) processes, respectively. "Non-zero covariance" refers to the case when the measurement errors and the innovations to the unobserved states are allowed to be correlated, "JOLTS hires" refers to the case when total hires are taken from the JOLTS database, "4-eq.model" is based on a specification from (12) with four observation equations, "2-st.model" is a two-state specification in (13) and "non-CRS" is a case when the assumption of constant returns to scale is relaxed. The benchmark model estimate is shown together with its 90% confidence bands indicated by the shaded area around the point estimate. The vertical shaded areas indicate NBER recessions.

3.4 An "over-identified" model specification

In principle, it is possible to estimate the state-space model using four observation equations: three describing the flows into employment from the three labor market states and one describing total hires. All these are linked via the structure of the aggregate matching function. In particular, the state-space system of this "over-identified" model specification can be written as:

$$
\begin{aligned}
h_t =& \mu\widetilde{s}_t + (1-\mu)v_t + \epsilon_t^h \\
h_t^u =& m_t^U + u_t + (1-\mu)(v_t - \widetilde{s}_t) + \epsilon_t^u \\
h_t^e =& \phi_t^E + e_t + (1-\mu)(v_t - \widetilde{s}_t) + \epsilon_t^e \\
h_t^i =& \phi_t^I + i_t + (1-\mu)(v_t - \widetilde{s}_t) + \epsilon_t^i \\
\widetilde{s}_t =& \log(M_t^U U_t + \Phi_t^E E_t + \Phi_t^I I_t)
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
m_t^U =& m_{t-1}^U + \eta_t^u, \\
\phi_t^E =& \phi_{t-1}^E + \eta_t^e, \\
\phi_t^I =& \phi_{t-1}^I + \eta_t^i,
\end{aligned}
$$

The results from estimating the above model are virtually identical to the benchmark specification with an elasticity of matching of 0.75 (0.025) and a matching efficiency closely tracking that of the benchmark (see Figure 1).

3.5 A two-state system

The benchmark model treats the employed and the inactive job seekers separately. To check the robustness of the results with respect to this choice, the following paragraphs describe and estimate a model in which the non-unemployed are treated as a single group. In particular, the state-space system for this two-state system can be written as

$$
\begin{aligned}
m_t =& \mu\widetilde{s}_t + (1-\mu)v_t + \epsilon_t^T, \\
f_t^U =& m_t^U + (1-\mu)(v_t - \widetilde{s}_t) + \epsilon_t^U, \\
\widetilde{s}_t =& \ln(M_t^U U_t + \Phi_t^O O_t)
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
m_t^U =& m_{t-1}^U + \eta_t^U, \\
\phi_t^O =& \phi_{t-1}^O + \eta_t^O,
\end{aligned}
$$

where $O_t = E_t + I_t$ are the group of non-unemployed individuals. The results from the above alternative model are very similar to the benchmark. In particular, the matching elasticity with respect to the number of job seekers is estimated at 0.78 (0.056). The estimated matching efficiency also closely tracks that of the benchmark (see Figure 1).

3.6 Relaxing the constant returns to scale assumption

The benchmark specification assumes that the matching function exhibits constant returns to scale (CRS). While this is a convenient feature (both empirically and theoretically), it remains a question whether it holds in the data. When job seekers outside of unemployment are ignored, it has been typically documented that CRS cannot be rejected at the conventional level of significance (see e.g. Borowczyk-Martins, Jolivet, and Postel-Vinay, 2013).

To relax the assumption of constant returns to scale, let $\mu_v = 1-\mu+\delta$ be the elasticity of matching with respect to vacancies. Testing whether δ is statistically different from zero provides a direct test of the CRS assumption. The results of this extension are very close to the benchmark values with $\mu = 0.726$ (0.058). However, the assumption of constant returns to scale can be rejected at the 1 percent level of significance since $\delta = 0.044$ (0.01). Nevertheless, the deviation from constant returns to scale is so small that it hardly affects the time-path of estimated match efficiency (see Figure 1).

3.7 Allowing for trends

This subsection investigates how the results change when deterministic trends are allowed in the latent variables. In particular, all unobserved states are assumed to follow the below law of motion

$$x_t = \overline{x} + x_{t-1} + \eta_t,$$

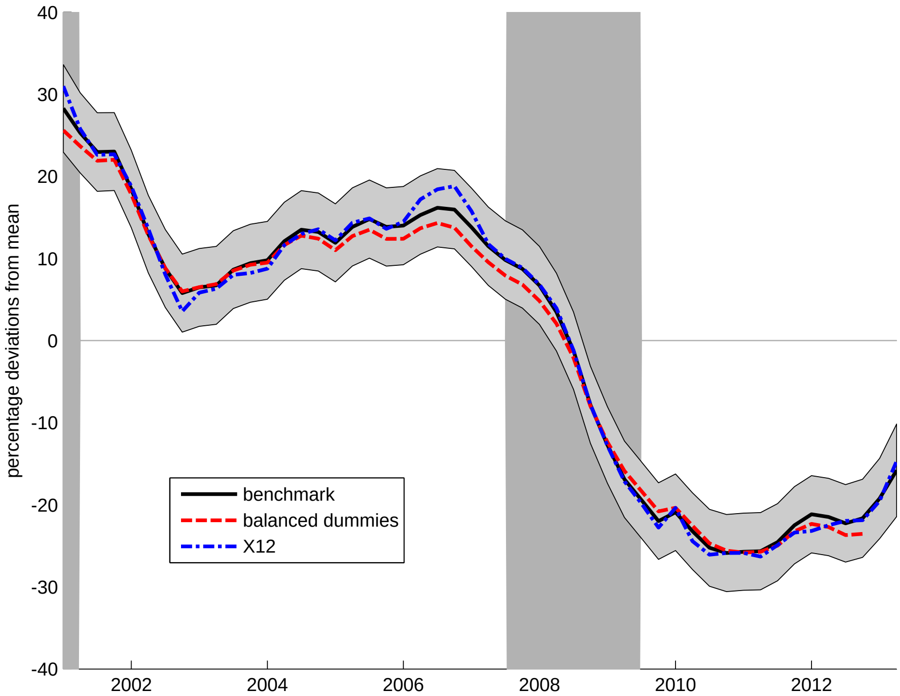
where x indicates the particular latent variable, η_t its innovations and $\overline{x}$ its trend. Table 1 shows that the results are essentially unchanged compared to the benchmark. This holds also for the time-path of matching efficiency which is almost identical to the benchmark (not shown).

4 Seasonal adjustment

Because the data used in the estimation is not seasonally adjusted, the benchmark model includes monthly dummy variables in all three observation equations. This section describes the results based on alternative ways of dealing with the seasonal pattern in the data. Before reporting these results, note that estimating the benchmark model on unadjusted data results in regression errors which violate the assumptions of normality, homoscedasticity and no serial correlation.

First, the benchmark sample starts in December 2000 and ends in June 2013. Hence, there is an unbalanced number of months and the estimated seasonal dummies may be affected by this.

Figure 2: Match efficiency of the unemployed: different seasonal adjustment



Notes: match efficiency estimates based on the 'benchmark" and alternative specifications: "balanced dummies" refers to the case when the benchmark model is estimated on a sample from January 2001 to December 2012 and "X12" refers to the case when the benchmark model is estimated using data adjusted for seasonality via the X12ARIMA method prior to estimation. The benchmark model estimate is shown together with its 90% confidence bands indicated by the shaded area around the point estimate. The vertical shaded areas indicate NBER recessions.

Second, we can simply use one of the existing methods of adjusting for seasonality *prior* to estimation. Here, I choose to seasonally adjust using the X12 method.

Both alternative cases produce very similar results to those of the benchmark model. In particular, the matching elasticity estimates are 0.729 (0.03) and 0.782 (0.043) for the balanced dummy variable and prior seasonal adjustment approach, respectively. Figure 2 shows the estimated matching efficiency for the benchmark and for the two alternative ways of seasonal adjustment. Again, the results are very similar in all three cases.

5 Measurement error and direct use of relative flows

Section 3.3 describes how the model is able to identify the unobserved states. In particular, the ratio of the fraction of effective job seekers in employment (inactivity) and the matching efficiency of the unemployed, $\frac{\Phi^E}{M^U}$ $\left(\frac{\Phi^I}{M^U}\right)$, is pinned down by systematic variation in the ratio of the probability of finding a job from employment (inactivity) and unemployment, $\frac{H^E/E}{H^U/U}$ $\left(\frac{H^I/I}{H^U/U}\right)$. The benchmark model adds measurement error to these relative probabilities as the unobserved (fundamental) states are likely to be smoother than the (relatively noisy) data. Nevertheless, this subsection reports that the results do not change substantially when the relative probabilities are taken at face value without measurement error.

Notice that once we assume that the relative probabilities are measured without error, it is possible to rewrite the model with only one unobserved state and it is no longer necessary to use a non-linear Kalman filter. In particular, we can estimate the following (linear) system

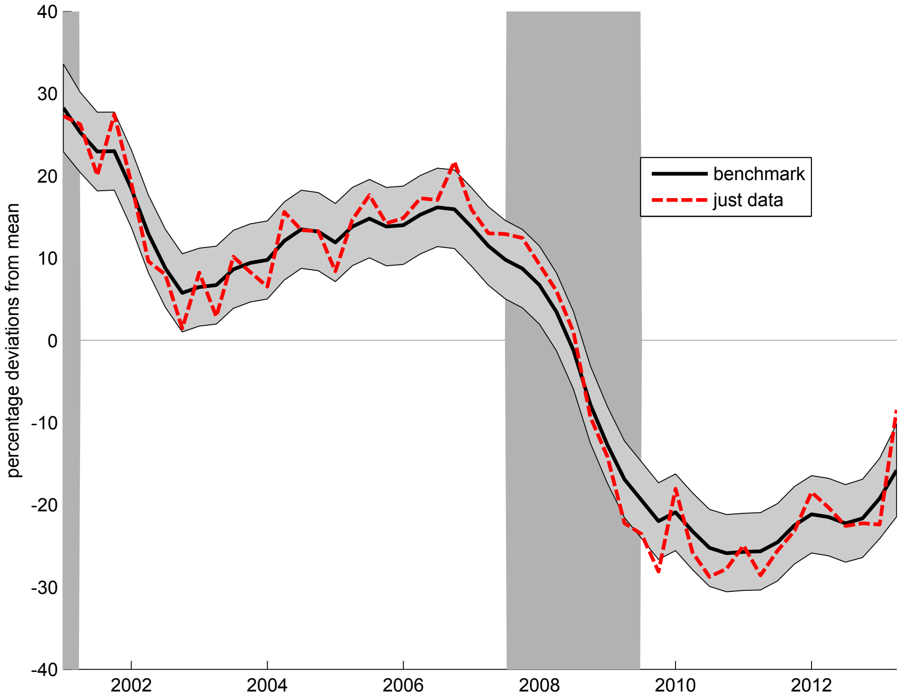
$$
\begin{aligned}
h_t =& \mu \widetilde{s}_t + (1-\mu) v_t + \epsilon_t^h \qquad (14) \\
\widetilde{s}_t =& \log \left(M_t^U \left[U_t + \frac{F_t^e}{F_t^u} E_t + \frac{F_t^i}{F_t^u} I_t \right] \right) \qquad (15) \\
m_t^U =& m_{t-1}^U + \eta_t^u, \\
\phi_t^E =& \phi_{t-1}^E + \eta_t^e, \\
\phi_t^I =& \phi_{t-1}^I + \eta_t^i,
\end{aligned}
$$

where the relative probabilities $\frac{F_t^e}{F_t^u}$ and $\frac{F_t^i}{F_t^u}$ were used to directly substitute out for the (previously assumed) unobserved states $\frac{\Phi^E}{M^U}$ and $\frac{\Phi^I}{M^U}$, respectively. Finally, $\epsilon_t^h \sim N(0, R)$ and $\eta = (\eta_t^u, \eta_t^e, \eta_t^i)' \sim N(0, Q)$.

Note that it is important to model the covariance between the innovations of the unobserved states as it imposes restrictions on the time-paths of the latent variables. This is not unrealistic as the three degrees of mismatch are likely to respond to several common driving forces and the observed relative job finding rates give additional information about the specific time-path of mismatch of the unemployed. Indeed, the benchmark model estimates the covariances between the innovations of the unobserved states to be strong (the correlation coefficients between the innovations to match efficiency and the other two latent variables are -0.72 and -0.96, respectively). The next section uses a Monte-Carlo exercise to document that if the correlation between the state innovations is not zero, then ignoring it leads to biased estimates of the unobserved states and the matching function.

The results based on estimating (14) are similar to the benchmark. The only noticeable difference is that now matching efficiency is, perhaps unsurprisingly, estimated to be noisier (see Figure 3). This is also reflected in the estimate of the matching elasticity μ which is now estimated much less precisely. The point estimate is 0.79 with standard error of 0.14. Moreover, apart from the noisier estimates, this specification suffers from serial correlation in the forecast errors. The assumption of no serial correlation is rejected at the 1 percent level of significance.

Figure 3: Match efficiency of the unemployed: alternative measurements



Notes: match efficiency estimates based on the 'benchmark" and the case when $\frac{\Phi^E}{M^U}$ $\left(\frac{\Phi^I}{M^U}\right)$ is directly replaced by $\frac{H^E/E}{H^U/U}$ $\left(\frac{H^I/I}{H^U/U}\right)$ ("just data"). The benchmark model estimate is shown together with its 90% confidence bands indicated by the shaded area around the point estimate. The vertical shaded areas indicate NBER recessions.

6 Monte Carlo simulations

This subsection evaluates the estimation procedure within two Monte Carlo (MC) simulation exercises. The first exercise is aimed at understanding whether the benchmark state-space system in (11) can identify the latent variables. The second exercise shows that if one estimates the system without measurement error as in (14) it is important to allow the latent match efficiency to incorporate information from the (observed) shares of effective job seekers in employment and inactivity.

6.1 Performance of the benchmark model

To understand how well the benchmark model is able to capture the (true) unobserved states, the benchmark state-space model in (11) is taken as the data-generating process. In particular, the total number of (effective) job seekers is constructed by using the estimated smoothed match efficiencies from the benchmark model and data on the stocks of unemployed, employed and inactive individuals. Then, this time series is combined with data on vacancies according to the structure in (11) and with $1,000$ draws of $\epsilon_t \sim N(0, R)$, where R is the estimated variance of the measurement errors in the benchmark model. This generates $1,000$ time-paths of "artificial" data on total hires, unemployment-to-employment and employment-to-employment transitions which used to repeatedly estimate the state-space system in (11).

Figure 4 shows the three unobserved states estimated in the benchmark model together with the respective averages over the $1,000$ MC simulations (the shaded areas depict the 90% bands, i.e. within those bands lie 90% of the estimated smoothed states).[2] The estimation procedure clearly does a good job at picking up the variation in the unobserved components.

6.2 Estimating without measurement error

As was mentioned in Section 5, when estimating the model without measurement error in which match efficiency is the only unobserved state, it is important to allow it to incorporate information from changes in the the share of effective job seekers from employment and inactivity. The reason is that in the benchmark model these have been estimated to be relatively high (correlation coefficients between the innovations to match efficiency and the other two latent variables are -0.72 and -0.96, respectively).

This subsection shows that severing this structure results in a loss of important information on the time-path of the latent variable and leads to biased results. Towards this end, I construct artificial data using the following state space system

$$\begin{aligned} f_t =& m_t + \mu(v_t - s_t) + \epsilon_t, \\ s_t =& x_t, \end{aligned} \tag{16}$$

$$\begin{aligned} m_t =& m_{t-1} + \eta_{m,t}, \\ x_t =& x_{t-1} + \eta_{x,t}, \end{aligned}$$

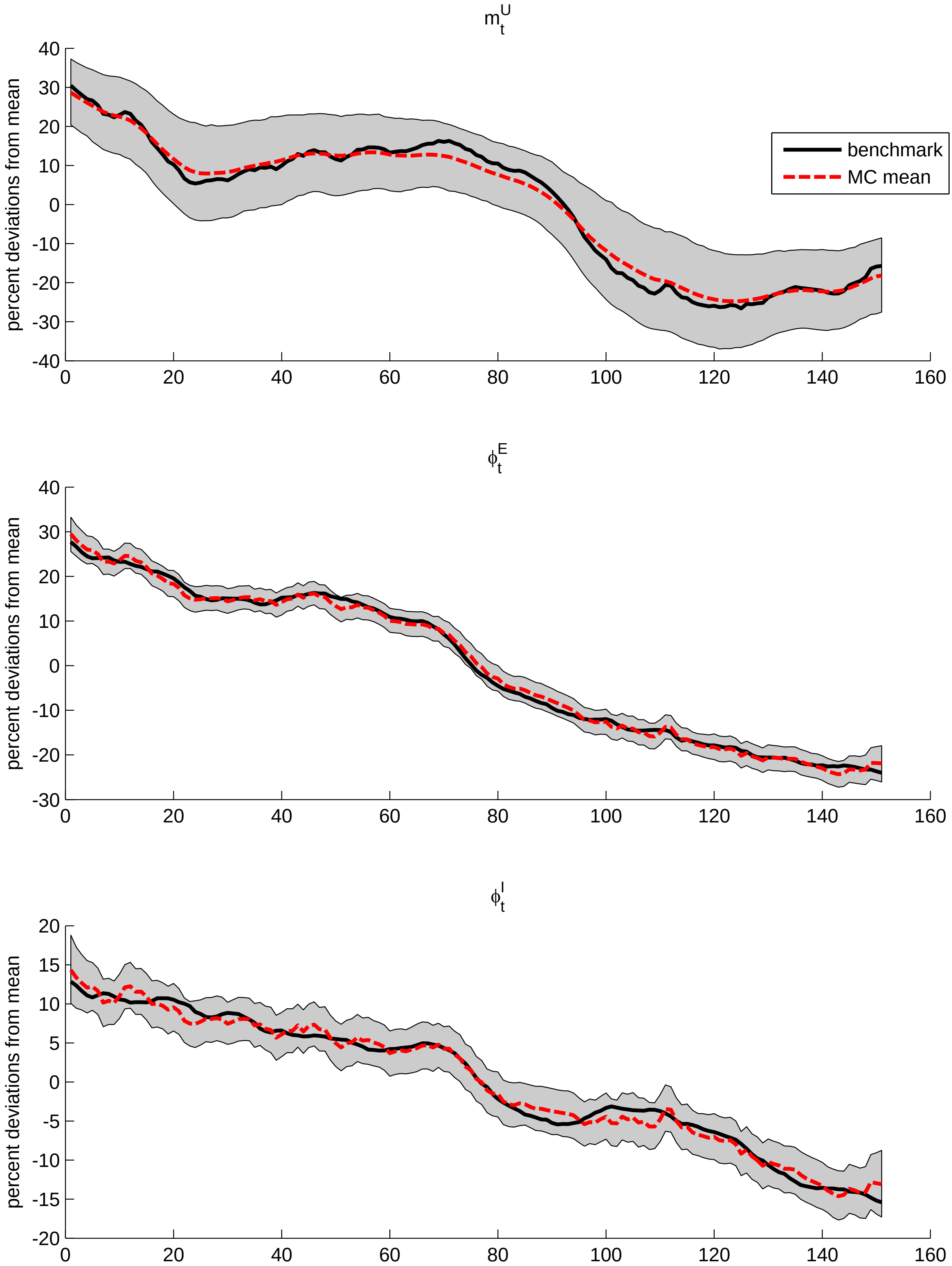
where $(\eta_{m,t}, \eta_{x,t})' = \eta \sim N(0, Q)$ and $\epsilon_t \sim N(0, R)$. To parametrize the model, I use the actual variance of the measurement error in the job finding rate estimated in (13) to set R and I let $\mu = 0.75$. I set $Q(1,1)$ and $Q(2,2)$ to the variances estimated in (13) and importantly, I let the correlation between the innovations to the two unobserved state be equal to -0.8, the midpoint between the estimates in the benchmark model.

Next, I use the above system to generate a time-path for the two unobserved states. I then use these as the "true" unobserved states and generate $1,000$ draws of data on f_t and s_t using the actual data on vacancies and random draws of the measurement errors.

Finally, I use these $1,000$ datasets to estimate the above state-space system under two

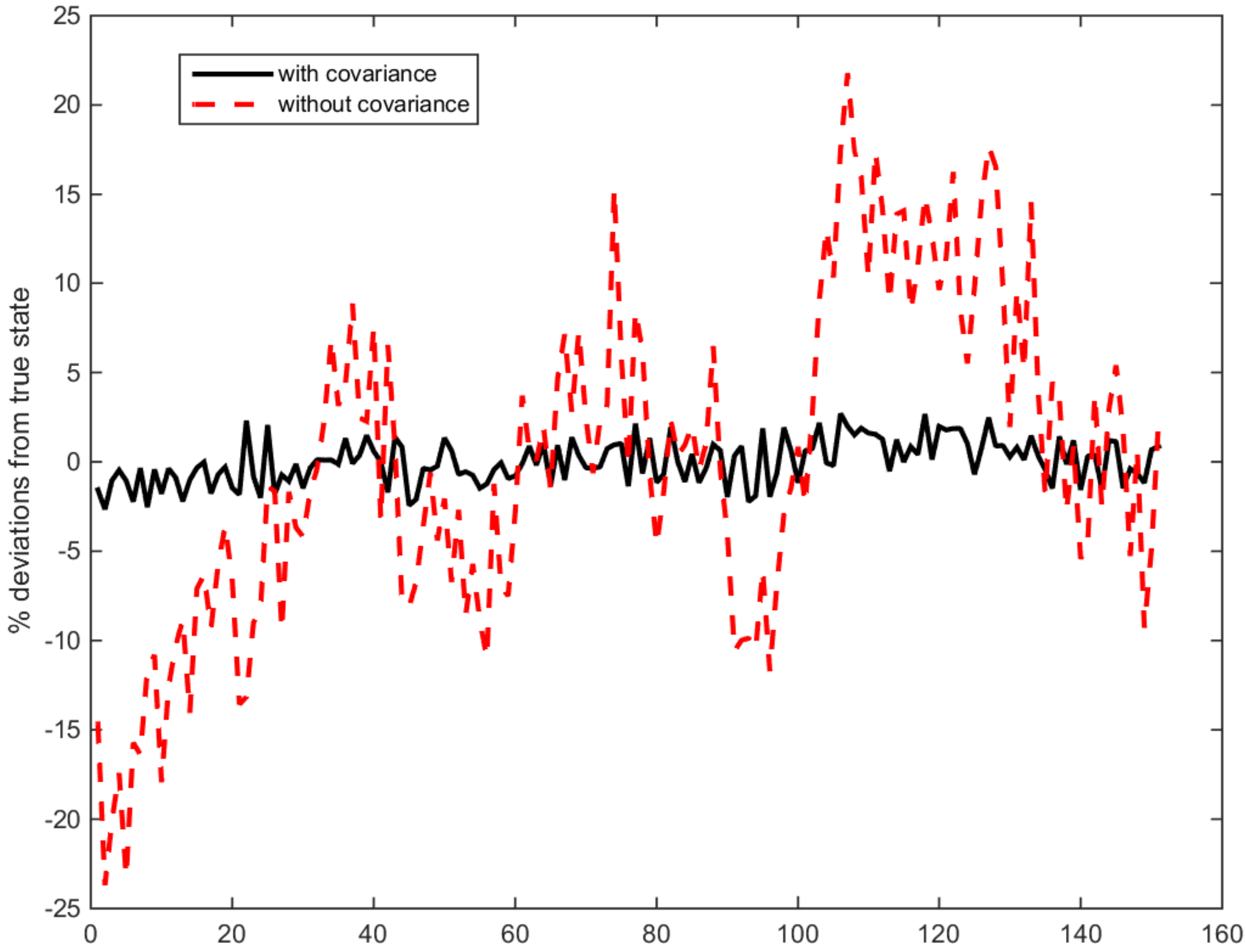
[2]The results are very similar if one considers medians instead of MC averages.

Figure 4: Match efficiency estimates: MC simulations



Notes: median match efficiency estimates across 1,000 MC simulations ("MC median") together with the true time-paths ("benchmark") for the unemployed ("M^U"), employed ("Φ^E") and inactive ("Φ^I"). The shaded areas are the 90% bands of the smoothed states.

Figure 5: Estimated deviations from true match efficiency: averages over MC simulations



Notes: difference between "true" and smoothed estimates of match efficiency averaged over $1,000$ Monte Carlo simulations. "With covariance" refers to the case when Q is allowed to be non-diagonal. "Without covariance" refers to the case when Q is restricted to be diagonal.

distinct assumptions. In the first case, innovations to the unobserved states are allowed to have non-zero covariance. In the second case, in the above model I restrict Q to be diagonal.

The results are plotted in Figure 5 which shows the percentage deviations of the smoothed states under the two specifications from the "true" unobserved state averaged over the $1,000$ MC simulations. Clearly, allowing Q to be non-diagonal is important in obtaining the correct time-path for the unobserved state. Ignoring that information leads to large deviations of the smoothed states from the truth.

These large differences from the true evolution of the unobserved states are also reflected in the estimates of the matching elasticity μ. While in the case when Q is non-diagonal the estimate of μ averaged over the $1,000$ MC simulations is 0.77 with a standard deviation of 0.03. The case when Q is restricted to be diagonal results in an average estimate of 1.07 and standard deviation 0.03.

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