Appendix to

Match efficiency and firms' hiring standards

Not for publication

Petr Sedláček[a,*]

[a] *University of Bonn, Adenauerallee 24-42, 53113 Bonn, Germany.*

A1. Derivation of the simple search and matching model

The economy is populated by a continuum of identical workers and firms. Firms hire workers on a frictional labor market in order to produce.

Matching of workers to firms occurs at the end of the period and matched workers are available for production in the next period. Let u_t be the mass of unemployed workers available for matching and let v_t be the mass of vacancies being posted by firms at the end of period t. The number of matches in period t is determined by a matching function

$$M_t = m_t u_t^{\mu} v_t^{1-\mu}. \tag{A1.1}$$

where m_t is a match efficiency shock serving the purpose similar to a Solow residual in a production function. The probability that a worker finds a job is defined as $f_t = M_t/u_t$, while the probability that a firm finds a worker for an open vacancy is $q_t = M_t/v_t$.

In what follows I describe the value functions of workers with and without a job and firms with a (un)filled vacancy. Denote with W_t the value at time t of being in a productive employment relationship for a worker. This is given by

$$W_t = w_t + E_t\left[\beta(1-\rho_x)(W_{t+1} - U_{t+1}) + U_{t+1}\right], \tag{A1.2}$$

*Tel.: +49 228 739 236

Email address: sedlacek@uni-bonn.de (Petr Sedláček)

where w_t is the wage rate, β is the discount factor and ρ_x is the exogenous separation rate. Hence, workers get a wage rate plus the continuation value of exiting period t in an employment relationship.

The value of being in the matching pool for the worker U_t at time t is defined as

$$U_t = b + E_t \left[\beta f_t (1 - \rho_x)(W_{t+1} - U_{t+1}) + U_{t+1} \right], \tag{A1.3}$$

where b is the value of leisure. Denote with J_t the value of a productive employment relationship for the firm employing a worker. This value is given by

$$J_t = z_t - w_t + E_t \left[\beta (1 - \rho_x)(J_{t+1} - V_{t+1}) + V_{t+1} \right], \tag{A1.4}$$

where z_t is output (aggregate productivity). The firm gets profits from production plus the continuation value of leaving the period in an employment relationship.

The value of an unfilled vacancy V_t is driven down to zero due to the assumption of free entry of firms. This then gives the vacancy posting condition

$$\frac{\kappa}{q_t} = E_t \left[\beta (1 - \rho_x) J_{t+1} \right], \tag{A1.5}$$

where vacancies are being posted until the expected future payoffs exactly equal the effective costs (κ / q_t).

Wages are assumed to be set according to Nash bargaining and are thus such that $(1 - \eta) W_t - U_t = \eta J_t$, where η is the bargaining power of workers. Using (A1.2) to (A1.5) one can obtain the following expression for the wage

$$w_t = \eta (z_t + \kappa \theta_t) + (1 - \eta) b, \tag{A1.6}$$

where $\theta_t = v_t / u_t$ is labor market tightness. The wage rate is a weighted average of firms' revenues and savings on hiring costs and the foregone outside option, where the weights are determined by the relative bargaining strengths.

Normalizing the labor force to 1, the law of motion for unemployment is given by

$$u_t = (1 - f_{t-1}(1 - \rho_x)) u_{t-1} + \rho_x (1 - u_{t-1}). \tag{A1.7}$$

Finally, the aggregate productivity shock and the match efficiency shock are assumed to follow first-order Markov processes

$$\log(z_t) = (1-\rho_z)\overline{z} + \rho_z \log(z_{t-1}) + \omega_{z,t}, \tag{A1.8}$$
$$\log(m_t) = (1-\rho_m)\overline{m} + \rho_m \log(m_{t-1}) + \omega_{m,t}, \tag{A1.9}$$

where $\overline{z}$, $\overline{m}$, ρ_z and ρ_m are the respective unconditional means and autoregressive coefficients. The innovations are iid normally distributed with zero means and standard deviations σ_z and σ_m, respectively.

Substituting in for wages and using the definitions of the job and worker finding probabilities, the model boils down to the following four equations

$$u_t = (1 - m_t\theta_{t-1}^{1-\mu}(1-\rho_x))u_{t-1} + \rho_x(1-u_{t-1}), \tag{A1.10}$$
$$\frac{\kappa}{m_t\theta_t^{-\mu}} = \beta(1-\rho_x)E_t\left[(z_{t+1}-b)(1-\eta) - \eta\kappa\theta + \frac{\kappa}{m_{t+1}\theta_{t+1}^{-\mu}}\right], \tag{A1.11}$$
$$\log(z_t) = (1-\rho_z)\overline{z} + \rho_z \log(z_{t-1}) + \omega_{z,t}, \tag{A1.12}$$
$$\log(m_t) = (1-\rho_m)\overline{m} + \rho_m \log(m_{t-1}) + \omega_{m,t}. \tag{A1.13}$$

A2. Details on the state-space specification and estimation

The state-space representation of the solution to the model can be written as

$$\mathcal{X}_t = \Phi\mathcal{X}_{t-1} + \Psi\epsilon_t, \tag{A2.1}$$
$$\mathcal{Y}_t = \Theta\mathcal{X}_t + \eta_t, \tag{A2.2}$$

where Φ and Ψ are coefficient matrices, typically non-linear functions of the structural parameters, ϵ_t is a vector of the two structural shocks (productivity and match efficiency), Θ is a selection matrix mapping model variables to observables and η_t is a vector of measurement errors which is non-zero when the number of observables exceeds the number of structural shocks.

The estimation is conducted using 4 variables: real GDP (y), Shimer's job finding rate (f^{Sh}), the CPS-based job finding rate (f^{CPS}) and the vacancy yield (q). The linearized equations for these variables, which then form the appropriate elements of the selection matrix Θ are the following

$$\hat{y}_t = (1-\overline{u})\overline{z}/\overline{y}\hat{z}_t - \overline{zu}/\overline{y}\hat{u}_t, \tag{A2.3}$$

$$\hat{f}_t^{Sh} = \hat{m}_t + (1-\mu)\hat{\theta}_t, \tag{A2.4}$$

$$\hat{f}_t^{CPS} = \hat{m}_t + (1-\mu)\hat{\theta}_t + \eta_t^f, \tag{A2.5}$$

$$\hat{q}_t = \hat{m}_t + (\mu_u + \mu_v)\hat{u}_t + \mu_v\hat{\theta}_t + \eta_t^q, \tag{A2.6}$$

where variables with bars indicate steady state values and hatted variables indicate log deviations from steady state values. As is explained in the main text all model parameters are calibrated using a standard procedure except for those related to the shock processes and the matching function which are estimated. The given parameters are estimated by maximum likelihood (ML) using Chris Sims' csminwel algorithm. The following values are used to initialize the minimization routine: $\rho_z = \rho_m = 0.9$, $\sigma_z = \sigma_m = 0.01$ and $\mu = \mu_u = -\mu_v = 0.5$. The initial values for σ_f and σ_q are set to the standard deviation of the difference between f^{Sh} and f^{CPS}.

Furthermore, to start the Kalman filter routine one must set the initial state vector $\mathcal{X}_0$ and its covariance matrix $P_{\mathcal{X},0}$. Following Durbin and Koopman (2001) the former is set to the unconditional mean of the state vector, while the latter is set to the identity matrix times a large number (10^5). This essentially means that there is large uncertainty about the initial state and the data is allowed to speak freely.

The estimated parameters are reported in the main text in Table (1). All parameters are estimated with a reasonable amount of precision, less so for those related only to the shorter time series. The elasticities on unemployment and vacancies in the vacancy yield equation, μ_u and μ_v, do not support a constant returns to scale matching function, but they do have the expected signs. Finally, the standard deviations on the measurement errors, σ_f and σ_q, are relatively large. However, as is shown below, estimating the model without them (only using data on real GDP and Shimer's job finding rate) delivers very similar results. Therefore, it does not seem that they drive the results.

A3. Estimating on different sub-samples

Figure (A7.1) shows match efficiency estimates based on different sub-samples starting in 1968 and 1985. All the estimates are very close to each other.

A4. Estimating with only Shimer's job finding rate

Figure (A7.2) shows the benchmark match efficiency estimate and that which is obtained by using only data on real GDP and Shimer's job finding rate. The estimates are virtually the same, but the estimate which uses only Shimer's job finding rate is 10% more volatile. This is consistent with Elsby et al. (2011) who document that Shimer's job finding rate fell more than the CPS-based measure which would be picked up by a larger match efficiency drop.[1] The similarity of the results also suggests that the measurement errors that need to be present in the larger state-space model in the main text are not driving the results.

A5. Comparison with a "model-free" estimation procedure

In an earlier version of this paper I use a "model-free" specification in which I estimate the matching function using only data on Shimer's job finding rate, unemployment and vacancies from the JOLTS database. I treat match efficiency and vacancies (as the JOLTS vacancy data starts only in 2001Q1) as unobserved and back them out using the Kalman filter. Figure (A7.3) compares that estimate with the one in the main text. Both estimates are very similar, except for a few periods around 2001, at the end of the sample.[2]

A6. Wage bargaining

In this subsection I describe wage bargaining in the presence of endogenous separations and firing costs. Workers coming from the unemployment pool do not posses any contract with the firm from the previous period. Therefore, if they do not come to an agreement with the firm over the wage, no firing costs need to be paid. Assuming Nash bargaining, the wage of newly matched workers is then a solution to $(1-\eta)(W_{i,t}^N - U_t) = \eta J_{i,t}^N$. On

[1] They relate this difference to misreporting of the duration of inflows into unemployment.

[2] The earlier version of the paper can be found at `www.psedlacek.com/Documents/Working/MeffOld.pdf`.

the other hand, when the firm decides to fire a worker that has been in an employment relationship in the previous period it must pay firing costs. The wage of a worker in an existing employment relationship is then a solution to $(1-\eta)(W_{i,t}^{E}-U_t) = \eta J_{i,t}^{E}+\phi)$. Using (14) to (17) in the main text, one can show that the wages of newly hired workers and workers in existing relationships are, respectively

$$w_t^N(p_{i,t}) = \eta(z_t p_{i,t} - \beta(1-\rho_x)\phi + \kappa\theta_t) + (1-\eta)b, \tag{A6.1}$$

$$w_t^E(p_{i,t}) = \eta(z_t p_{i,t} + (1-\beta(1-\rho_x))\phi + \kappa\theta_t) + (1-\eta)b, \tag{A6.2}$$

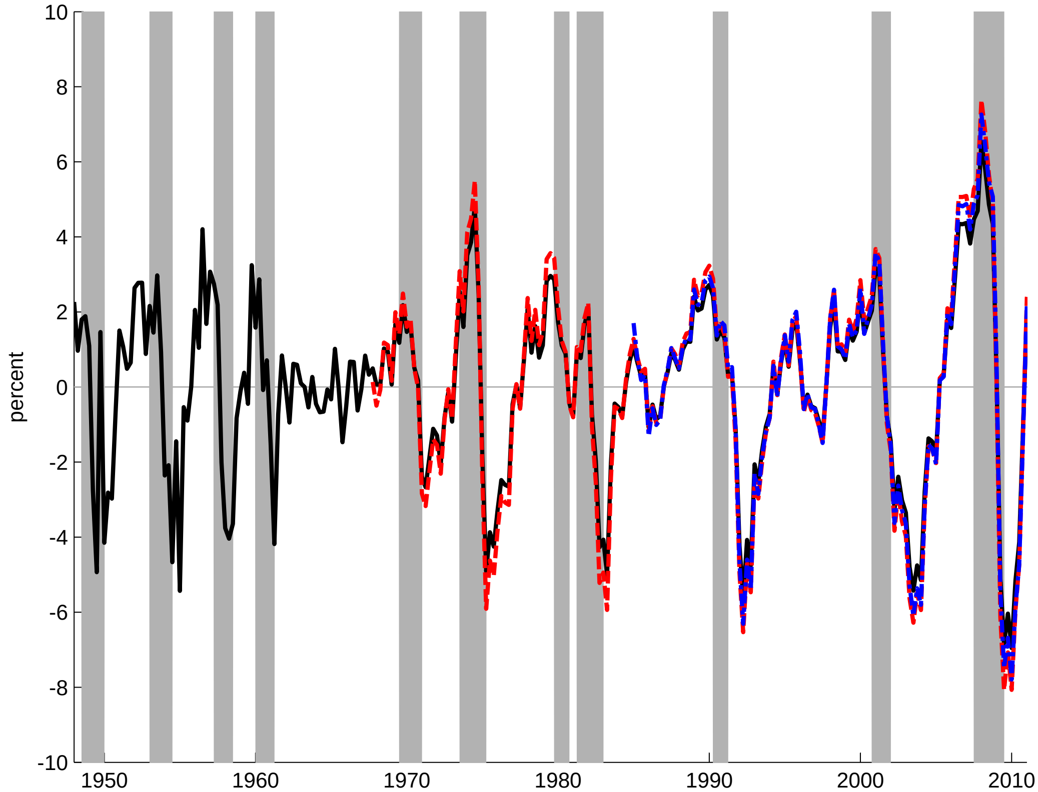
where setting $\phi = 0$ delivers the familiar wage structure in a standard model without firing costs. Newly hired workers are penalized because of the threat of having to pay firing costs in the future. On the other hand, workers in existing employment relationships now have a higher wage compared to the case without firing costs, because their effective bargaining power increased, since firing them entails a cost for the firm.

A7. Impulse response functions related to worker productivity

Figure (A7.4) shows the impulse response functions of variables related to idiosyncratic worker productivity to a negative one-standard deviation shock to aggregate productivity. In a recession both productivity thresholds rise (middle panel) as firms are willing to hold on to (hire) only relatively more productive workers. This is reflected by an increase in both the separation and rejection rates (top panel). However, as is discussed in the main text, the productivity threshold of new workers is in an area of the distribution with greater mass and therefore the increase in the rejection rate is considerably stronger.

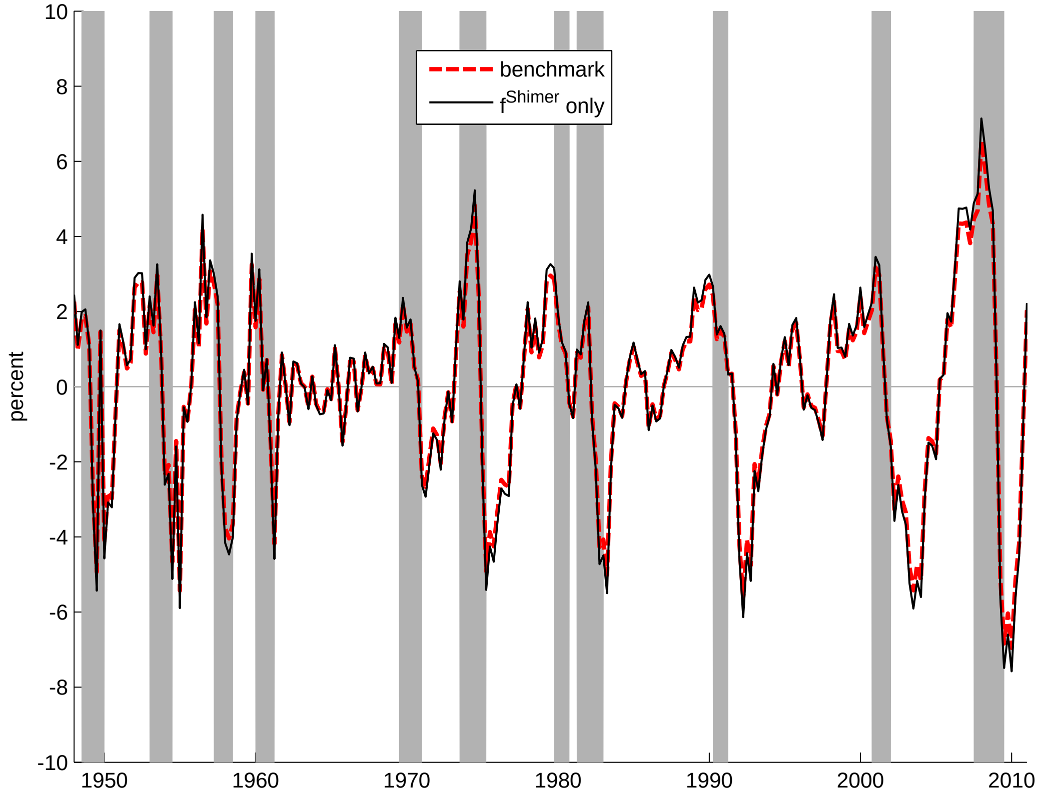
The bottom panel shows the increase in average worker productivity for both new and existing workers. Because of the stronger rise in the rejection rate, also average worker productivity increases relatively more for new workers. However, as was pointed out in the main text the strength of this increase is plausible.

Figure A7.1: Match efficiency, different sub-samples



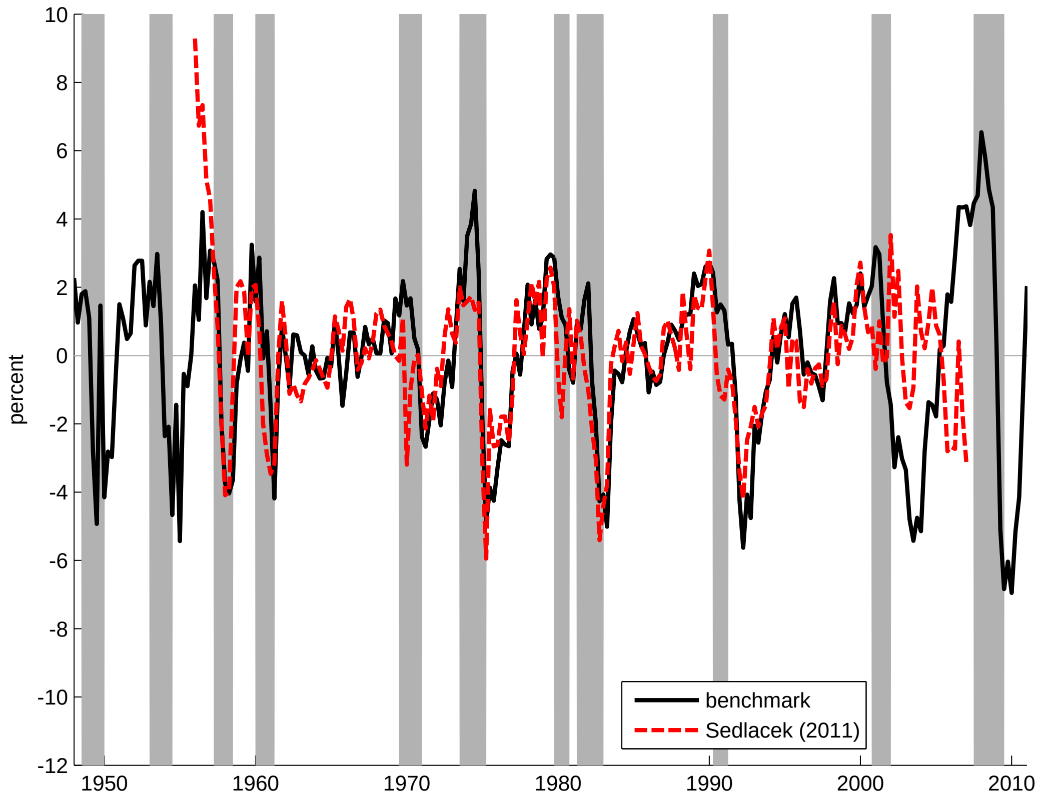
Notes: Match efficiency estimates based on the full sample, and sub-samples starting in 1968Q1 and 1985Q1.

Figure A7.2: Match efficiency, using only Shimer's job finding rate



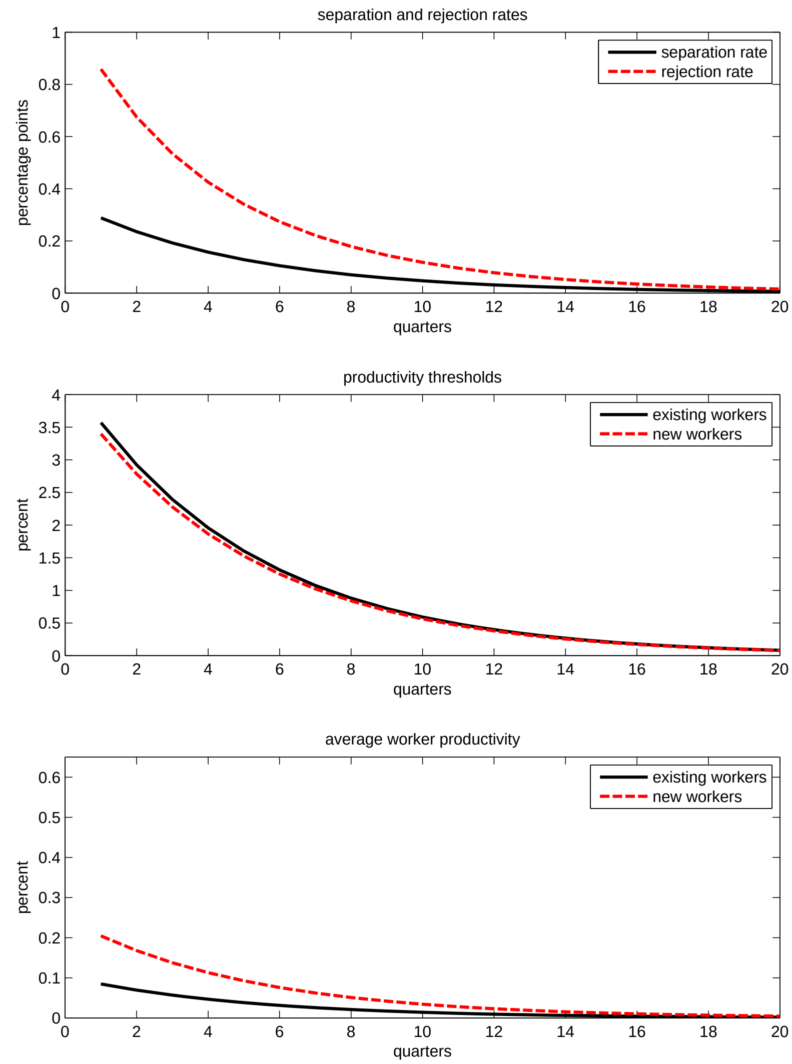
Notes: Match efficiency estimate using data only on real GDP and Shimer's job finding rate ("f^{Shimer}") and from the main text ("benchmark").

Figure A7.3: Match efficiency, benchmark and model-free specification



Notes: Match efficiency estimates from the main text ("benchmark") and a model-free estimate from an earlier version of this paper ("Sedlacek 2011").

Figure A7.4: Impulse response functions related to worker productivity



Notes: Impulse response functions to a negative one-standard deviation shock to aggregate productivity. The top panel plots total separation and rejection rates, the middle panel plots the productivity thresholds for existing ($\widetilde{p}^E$) and new workers ($\widetilde{p}^E$) and the bottom panel plots average productivity of existing ($H(\widetilde{p}^E)$) and new workers ($H(\widetilde{p}^N)$).